

1982

A mathematical analysis of transformation-invariant cues in the recognition of simple shapes.

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A MATHEMATICAL ANALYSIS OF TRANSFORMATION
INVARIANT CUES IN THE RECOGNITION OF SIMPLE SHAPES

by

C

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A Thesis
Submitted to the Faculty of Graduate Studies
through the Department of Psychology
in Partial Fulfillment of the
Requirements for the Degree
of Master of Arts at the
University of Windsor

Windsor, Ontario, Canada

1981

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DEDICATION

This thesis is dedicated to Katharine, who married into it.

ABSTRACT

The transformation-invariant cues involved in the recognition of simple shapes were analyzed mathematically. A stationary and then moving arbitrary two-dimensional object, displayed in a fixed observer's frontal plane, was investigated and general formulae involving the angular height were developed. These results were then applied to an arbitrary rectangle, trapezoid, circle and ellipse.

The analysis indicated that whether an object is stationary or moving, the transformation cues may always be partitioned into two components. The first contains true shape and distance information, and the second, distortion due to slant. The difference between the static and moving object cases was found to arise from a discrepancy in the velocity terms contained in their respective distortion due to slant components.

ACKNOWLEDGEMENTS

The author wishes to extend his sincere thanks and appreciation to the chairman of his advisory committee, Dr. A. Arthur Smith, who has, with great proficiency and patience, guided this study through to its completion. The suggestions and contributions from the other committee members, Dr. Harold R. Atkinson and Dr. Martin Morf, have also been greatly appreciated.

The author wishes to thank Katharine Pashley for drafting the figures and proofreading the paper. Thanks also to Honey Robb who worked overtime to type the paper.

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CHAPTER I

INTRODUCTION

How does an observer distinguish between the change in position of an object and the change in state of an object? This was a psychological question asked by the French mathematician Poincare (1913). This question has yet to be answered by psychologists. By "position", Poincare referred to the object's spatial orientation in the environment. The observer's perception of this object may change if the individual or the object moves. By "state", he meant the identity of the object which can change when the object grows or changes shape.

For a long time, psychologists have been puzzled by what has been called the approximate constancy of visual things (Gibson, 1950a). Objects look much the same size whatever their distance from the observer, and the same shape at different angles of regard, or from different points of view.

This paper is an attempt to analyze, mathematically, the visual cues which lead to our recognition and discrimination between simple shapes, such as circles, ellipses, squares and trapezoids.

Originally, perceived form relates directly to the shape of the retinal image (Pastore, 1971). If the image is a square or circle, the original

perception is also a square or circle. Generally, the form of the original perception corresponds to the distorted perspective shapes of the image. Usually in adult perception, an object retains its form despite the distortion of the retinal image which is correlated with it. This particular fact defines the concept of shape constancy. For example, the perceived shape corresponding to a circle or square moved about in space is a circle or square, rather than an ellipse or trapezoid, respectively.

The common sense answer to this question of why we see an object as being possessed with constant dimensions from whatever position we view it, is simply that we know the object. In other words, we assume that we have handled it in the past, or if not, we at least know the laws of physics about all material objects, including unfamiliar ones.

William James (1887) stated that the ambiguous relationship between the retinal shape and the real shape of the external object was the basis for his empiristic approach. The constancy of size and shape are included among the perceptions that would be acquired. James concluded that in our dealing with an object, we always choose one of the infinity of sensations which constitutes its real form. For example, a table top yields an infinite number of retinal images, when viewed from different positions. Then, the mind chooses the square and ignores the perspective views.

Wundt (1896) said in respect to a child during his first year, that the perception of size, distance and various three-dimensional figures remains for a long time very imperfect. In particular, distant objects are all

thought to be near at hand, so that they appear relatively small to the child. As it was thought that shape constancy was learned, the form of the original perception would conform to the perspective changes of the retinal image. Earlier, Clifford (1873) had developed the idea that the imagination can fill in, so to speak, to correct the deficiencies of the original perception.

These familiar assumptions will explain a great many of our perceptions, including an object with constant dimensions. Whether it is necessary to call upon so intellectual a process to explain consistency in shape or form perception, though, is questionable (Gibson, 1950a).

Ames (1946) developed some demonstrations that showed the influence of past experience on the visual perception of ambiguous configurations (see Ittelson, 1952). In one situation, a distorted room was used. The subject looked through a small hole with one eye and saw a room having the dimensions 10 X 6 X 5 feet. Two windows were placed in the wall opposite the peephole. The subject was then told to place his hand in a small opening in the near wall, and grab a long pointer and with it touch the far right corner of the room near the ceiling. To the subject's surprise, upon doing this, he fell far short of touching the designated point. During the second stage of the demonstration, the subject was allowed to look into the room with both eyes while freely moving his head. He then, for the first time, saw the room in its distorted form. In previously regarding the room through the peephole, without prior knowledge of its shape, the subject depended on his past perceptual history. In such a room, the windows in the far wall were obvious reference configurations. When they

were taken to be of equal size, the pattern of perception for the whole room was set and was judged to be rectangular. In the later demonstrations when the subject viewed the room binocularly and from different viewpoints, he realized that the windows were not equal in size. In fact, all the former angular relations were then changed to provide an unambiguously different configuration. Ames (1946) showed that it is the characteristic of equivalent configurations to be perceived in many different ways. In such situations, the subject's interpretation depends on his past perceptual history.

The Perception of Form and Shape Constancy

The perception of form is an interesting study which provides a basis for the study of shape constancy. Even in a limited sense, there seems to be at least three general meanings of the term form (Gibson, 1951). First, there is the substantial shape of an object in three dimensions. The second is the projection of such an object on a flat surface. A drawing on canvas would be an example of this second type of form. Thirdly, there is the abstract geometrical form composed of imaginary lines, planes, or families of them. This last meaning of form would be the farthest removed from the real object.

A classical assumption commonly made is that two-dimensional vision is immediate while three-dimensional vision is secondary or perceptual (Gibson, 1951). In other words, one must first see a plane form before one can see a solid form. Gibson (1950b) has suggested that this argument is a fallacy. In fact, Gibson claims that solid vision is primary while plane

vision is actually secondary. Gibson (1950b) gives evidence to show that plane vision is acquired only with training and by adopting a special attitude.

Gibson (1951) defined outline form as physical tracings made with ink, pencil, or paint on a surface, which geometrically represent the edges of a surface-form or the margins of a solid form. The perception normally aroused by an outline-form is quite unlike the outline itself. Koffka (1935) showed that the paper surface appears to become the background and to recede while the enclosed surface seems to take on a figural quality and stand out.

Any given outline-form is only one of an infinite set of perspective transformations. Physically, when a surface-form is projected onto another surface by light, the differing orientation of the form to the surface yields a set of different perspective forms. Psychologically, when a surface-form is viewed at different angles or regard, the observer obtains a set of different perspective impressions. However, Gibson (1951) notes that a subject obtains a constant perception of the surface-form with a varying impression of slant.

Therefore, the problem of shape constancy will be solved when psychologists can explain how a given face of a solid object is seen to have a certain form at a certain slant. From this, the explanation of how we see the whole object in three dimensions can be derived (Gibson, 1951).

Gibson continues on to say that the primary problem for psychologists is to isolate the invariant properties in visual stimulation which are in psychophysical correspondence with constant phenomenal objects.

Experimentally, shape constancy is assessed when a subject estimates the shape of objects such as a disc or rectangular plate tilted or slanted at an angle (Schiffman, 1976). Generally, when a subject is shown an inclined circle and is asked to select one which represents the shape seen by him, he chooses without hesitation an ellipse. This ellipse, however, is widely different from the one which represents the shape of the inclined circle indicated by the laws of perspective, being much nearer to the circular form. The subject sees an inclined figure neither in its "real" shape nor in the shape which is its perspective projection but as a compromise between these.

Thouless (1931a) was first to show this phenomenon experimentally. His subjects viewed shapes of objects which were either viewed obliquely, with the apparent brightness of differently illuminated surfaces of different reflectivity, with the apparent sizes of objects at different distances or with the apparent convergence of parallel lines receding from the observer. In all of these cases, it was found that what was seen was intermediate between what was given in peripheral stimulation and the "real" character of the object. It was found that the estimated shape, obtained by having subjects draw the shape of a tilted disc or match it against a series of ellipses, was more circular than the elliptical shape projected on the observer's retinae. To this effect of the character of the "real" object on the phenomenal character, Thouless (1931a) gave the name phenomenal regression to the real object.

Early Theories of Shape Constancy

An early and highly elaborate theory of shape constancy was created by

Koffka (1935). Koffka's explanation of the phenomena of constancy of shape rests upon two assumptions, the first of which is that the frontal vertical plane is the most stable of all planes, and any surface not in that plane will tend to regress (phenomenally) towards it. Secondly, some objects have more highly organized, or more stable shapes than others; and hence there will be a tendency for any object to regress to a more highly organized shape. This second assumption is largely based on Eissler's (1933) results that shape constancy appears to be different for differently shaped objects.

Moore (1938) designed an experiment to test Koffka's theory, by measuring the constancy of shape for two different figures the straight line and the circle. No particular length of line is regarded by Gestalt psychologists as more highly organized than another. On the other hand, a circle is regarded as more highly organized than an ellipse. If the theory holds, that is, if internal organizing forces do play a part, then a greater degree of constancy ought to be shown by a tilted circle than a tilted line. The stimuli used consisted of vertical lines and ellipses. The phenomenal length of a straight line, tilted from the frontal view to various angles of tilt, was compared with shorter lines. A similar procedure was done for a tilted circle, by comparison with ellipses. The ellipses were constructed geometrically on white cardboard while the lines were constructed of carefully straightened galvanized iron wire. A number of inferences were made as a result of this experiment. The most important was that a tilted circle was found to show a greater degree of constancy than a tilted line. However, Moore (1938) suggested that the subjective method, and not internal organizing forces, was largely responsible for the difference.

Stavrianos (1945) made an effort to verify part of Koffka's theory also.

The hypothesis that perceived orientation combined with apparent shape yields a constant shape was tested. Also examined was the assumption that when the orientation was seen correctly, the constancy would be complete, and if the slant was not visible, there would be no constancy. In this experiment Stavrianos (1945) used tilted cardboard rectangles. He simply determined whether observers who failed to perceive the tilt correctly, also failed to perceive the tilted dimension by a geometrically equivalent amount. It was predicted that if an observer judged the degree of tilt as less than it was, he should then judge the rectangle as just so much shorter than it was. The experiments produced no conclusive evidence to support this prediction.

A theory of shape recognition was also presented by Deutsch (1955). He tried to explain through his theory, six basic facts, the first of which is that animals can recognize shapes independent of their location in the visual field. Secondly, recognition has been known to be affected independently of the angle of inclination of a figure in the visual field. The third fact is that the size of a figure, except at extremes, does not interfere with the identification of its shape. Fourthly, both rats and humans tend to confuse mirror images. The fifth fact states that primitive organisms find it hard to distinguish between squares and circles. And finally, the neurological fact that the above abilities, which appear to be mediated by the striate cortex, survive the removal of the major part of it.

This theory of form recognition is complementary to a previously published theory (Deutsch, 1955) of motivation and learning. A mechanism which displays the required properties to explain the above six facts is basic to his theory. Deutsch indicated that the mechanism was not entirely without neurophysical plausibility.

Experimental Methods

Thouless (1931b) in his early studies used two techniques for the measurement of shape constancy. The first method used was to simply ask an observer to make a drawing of a standard figure. The second method asked the observer to match a shape with one of a variable series of figures. For example, if the subjects were shown a tilted circle, they would be requested to match it with one of a number of ellipses presented in the frontal-parallel plane. The ellipses would be selected as to represent the retinal projections of the standard at different degrees of tilt.

Moore (1938) employed a less crude technique which permitted measurement by a continuous method. The observer was required to vary the orientation or slant of one figure until it appeared equal in shape to a second figure, the objective shapes which differed from that of the first. However, Gottheil and Bitterman (1951) claimed that Moore's method did not provide a meaningful index of shape-constancy.

A number of studies have shown that shape constancy is diminished by conditions that reduce the availability of effective cues to orientation (Epstein and Park, 1963). Thouless (1931a, 1931b) found that constancy was

consistently lower for monocular than for binocular observation. A disc lying on a table top became more elliptical when the observer switched from binocular to monocular vision.

In Thouless' (1931a, 1931b) experiments, a high degree of constancy did occur under monocular conditions, though less than binocular conditions. However, these results shifted when Thouless' experimental setting was darkened so that only the disc was visible. Afterwards the phenomenal shape equalled the retinal shape. Langdon (1951, 1955) also found poor constancy ratings for monocular viewing when cues to slant were eliminated by darkening the experimental room.


Lichte and Borresen (1967) looked at the influence of instructions on the degree of shape constancy. During a typical shape constancy experiment, subjects have basically two responses open to them. One is to indicate the shape of the stimulus object when optically projected upon a plane perpendicular to the subject's line of sight (the retinal image value). The second is a response indicating the shape of the stimulus object when it was placed perpendicular to the line of sight (the object value). Lichte and Borresen's (1967) study looked at the effects of three kinds of instructions given during a shape constancy task. The first two instructions referred to the two above described responses. The third, and least clearly defined required response, was to the perceived or apparent shape of the object. The results indicated that the instructions given to subjects do make an important difference in their responses. In particular, an object instruction produced a higher degree of constancy when compared with the other two types. The writers concluded that care should be taken when selecting instructions for shape constancy experiments.

Two years later, Hake and Myers (1969) conducted a study on the relationship between familiarity and shape constancy. They looked at the effect that shape familiarity of stimulus forms had in recognition tasks. Subjects selected from a series of perspective views of a tilted form, views matching a standard view which had been seen previously. Hake and Myers found that prior familiarity with a single perspective view produced a response effect which was an increase in overall recognition rates for all tilted views of that form. They claimed that only familiarity restricted to a radically tilted view of a form produced the constant error effect under image instructions commonly accepted as shape constancy. Hake and Myers argued that this information should be taken into account when involved with shape constancy experiments because of the nature of differential acuity for alterations in the appearance of plane forms due to the tilting variable.

A further study on the effects of both instructions and training was conducted by Gregg and Pasnak (1971). Subjects were instructed to judge both the true shapes and the retinal images of tilted figures in reduced cue viewing conditions. The observers were also given feedback on the correctness of their responses. The figures consisted of six standard triangles with equal bases, but varying altitudes. Gregg and Pasnak found that instructions were effective, as there was no overlap between retinal and constancy judgements. Also, there was a small, but significant improvement with training under constancy instructions. They concluded that subjects can change their judgements toward a retinal or a constancy match, depending on instructions, even in a situation where cues have been substantially reduced. A training procedure which allowed feedback also enhanced this effect.

Cues and the Estimation of Curvature

Cues have always served as a basis for discussing the problem of form perception. (Smith and Smith, 1957). These cues are customarily treated as single, isolated variables, even though the observer's optical array in his daily world is composed of combinations of them. Smith and Smith (1957) studied the interaction of the effects of cues involved in judgements of curvature. The hypothesis of the study stated that cues or variables interact in their effects upon judgements of the curvature of a three-dimensional curved object such as a cylinder. The judgement of curvature was used as a criterion because apparent continuity of the curvature, beyond the visible limits of the surface, was reported when the observer has a unrestricted view of the cylinder. This then furnished an index of normality of judged shape. The apparatus used was a semicylinder that was mounted on a central horizontal axis so that the observer's line of sight bisected the width and height of the surface and was perpendicular to its horizontal axis. Spring clips held removable paper jackets which smoothly fitted the exterior surface of the cylinder. Three patterns of markings on the removable paper jackets were used, consisting of 135, 45 and 9 black spots with an additional red fixation spot in the center of each. The use of different shapes minimized the effects of form constancy as cues to distance differences. Subjects were asked to describe the shape. The most important result (Smith and Smith, 1957) of this experiment was the significance of the first-order interactions. Judgements of curvature in this situation were not simple functions of the degree of binocular disparity. Smith and Smith also found that greater curvature was reported in binocular, rather than in monocular vision. Also, greater curvature was



reported for closer objects. Finally, interactions of cues in determining perceived distance were characteristic of judgements of curvature in this experiment.

In a more recent study, Dobson (1971) investigated size, convexity and curvature magnitude estimations of curvilinear forms. Subjects made magnitude estimates of size, convexity and curvature of patterns along a shape continuum. The experimental forms were generated mathematically and were presented to observers from 33 mm slides. The results suggested that psychophysical scales of curvature and convexity may be developed which are of no use experimentally. Also, the ambiguity of the estimation task for curvature and convexity is probably greater than for size (Dobson, 1971).

Working with circles, Virsu (1971) looked at task dependence and underestimation of curvature in visual perception of form. The apparatus used consisted of two sets of 12 test arcs with arc lengths from 9 to 342 degrees of the center angle, which were drawn on 20 X 20 cm. pieces of cardboard. A force-choice version of the method of constant stimuli was used. Subjects judged whether the circumference of the circle was less or more curved than the arc. The observers then indicated their judgements by placing the circle in one of two piles of comparison circles. The results suggested that estimates of the linear dimensions of arcs indicate overestimation of the curvature of short arcs. Virsu concluded that the overestimation of curvature found in earlier studies may have resulted from the use of inappropriate perceptual tasks.

Zusne (1975) designed four experiments to study the perception of curves. He was interested in the role played in visual perception by curvature, which is a dimension of visual form. In particular, Zusne looked at the position of curvature among other dimensions of visual form (e.g. is it an information or a configurational dimension). Also investigated was the question of whether the way visual form is processed may be affected by the locus of curvature. Finally, Zusne studied the parametric relationships between degree of curvature and response. Random polygons and their curvilinear transformations were presented to subjects. In all the experiments, curvature affected perception at statistically significant levels. The extent of this effect was a function of bows of curvature, the level of compactness of the figure and the nature of the perceptual task (Zusne, 1975).

Discrimination Between Familiar Shapes

A number of studies have been done in which subjects discriminate between circles and ellipses. Laursen and Rasmussen (1975) investigated circle-ellipse discrimination in both man and monkey. The aim of the study was to compare thresholds of perceptual discrimination of shape. Circles and ellipses were used in this particular study so that the task could be continuously graded and also because ellipses are easily defined by the ratio of their axes, while circles are even more conveniently measured. The human subjects were instructed and the monkeys were trained to press a panel with a black projected circle among seven panels with identical ellipses. Both human and monkey subjects discriminated circles from ellipses very well with about the same accuracy. The best performers of

both species chose 75 percent correct when the difference between the axes of the ellipse was the width of the contour or 2.5 percent. Both humans and monkeys performed equally well with horizontal and vertical ellipses. Chance performance that was measured when all stimuli were circles showed that extraneous cues were absent (Laursen and Rasmussen, 1974).

An interesting study using circles and ellipses was conducted by Wilde, Siegel and Williams (1967). They studied the recognition of lunar craters. Since the crater is the most common and consistent in shape of the lunar surface features, developing appropriate optical aids for lunar missions could be very useful. Hence, the image characteristics were needed for the visual recognition of lunar craters. For simplicity, the crater was considered as a simple circular form. The elliptical image measurements for various sized craters were developed with the help of a General Precision R PC 4000 Computer. Due to the minute visual angles under consideration, calculations had to be carried out to eight significant places to obtain the desired level of accuracy. These image measurements were then combined with some previous threshold recognition data for the ellipse (Casperson, 1950). The elliptical image measurements consisted of the visual angle of the major axis and elliptical form. Recognition thresholds were determined at an orbital altitude of 80 nautical miles. By graphically combining the visual angle and form data, Wilde et al., computed 50 and 70 percent threshold recognition curves relating crater size, magnification and look angle.

A perceptual bias toward symmetry in shape constancy was studied by King, Meyer, Tangney and Biederman (1976). Subjects had to discriminate between

not only circles and ellipses, but also squares and rectangles. The apparatus used consisted of a display box, a projector as light source and a variable speed shutter. Each of the subjects were presented with a circle-ellipse series in which they saw three circles and three ellipses, each object for three different lengths of time and at three different angles of inclination and at all possible combinations of conditions for a total of 54 presentations. A similar square-rectangle series was also presented to each subject for a grand total of 108 trials per subject. The subjects entered their judgements and confidence ratings on a response sheet. A bias towards prevailing symmetry was evident when the subjects judged whether a briefly presented rotated figure was symmetrical (circle or square) or asymmetrical (ellipse or rectangle). This bias appeared to be perceptual rather than judgemental in origin. King et al. then proposed a model which incorporated both the similarity space and the symmetry. The model can account for the paradoxical finding that the more asymmetrical some projected images are, the more likely they will be perceived as symmetrical. The model states that tolerance for discrimination given some constant error or resolution of slant or height-to-width ratio is reduced as rotation increases (King et al., 1976).

An earlier study by Veniar (1948) was done on difference thresholds for shape distortion using squares and distortions of squares. An experiment was designed to discover the difference threshold for the perception of shape distortion in squares, and to determine how the difference threshold varies as illumination and size of the object varies. The distorted squares were rectangles resulting from an elongation or shortening of one dimension in equally spaced steps. Distortion appeared in both the

horizontal and vertical planes. The obtained average difference threshold in all sizes of square figures used was 1.4 percent of the original, invariable, stimulus side. Results also indicated that the different thresholds for shape distortion do not vary systematically as illumination varies (Veniar, 1948).

Attneave (1950) examined the question of what makes objects seem alike or different, in his paper on dimensions of similarity. Five experiments were reported. The first three involved judgements of similarity on pairs of stimuli varying multidimensionally. The other two experiments employed the same visual objects as in the first three, but used a new measure of similarity. The same procedural pattern was common to all. The subject was presented items in association with a short word. The subjects then attempted to reproduce the names of the stimuli, and the number of confusions between any two objects was taken as a measure of their similarity. Attneave compared the two measures of similarity that were used, and found that different dimensions may be given different weights by the two measures. Therefore, in the multidimensional situation, prediction of the confusions from judgements was not generally possible without separate consideration of the dimensions involved.

Degree of Orientation

Several investigators who have dealt with the degree of orientation, report that the amount of constancy expressed in terms of the Brunswick and Thouless ratios, or various other indices, does not remain constant for different slants. Sheehan (1938) found that constancy decreased as the

angle of rotation from the frontal-parallel plane increased. Lichte (1952) obtained a linear function between the Brunswick ratio and the angle of rotation of the stimulus object from the frontal-parallel plane. This finding that the Brunswick ratio decreased with angle of rotation was probably an artifact of Lichte's peculiar assignment of values (Epstein and Park, 1963).

The results obtained by Thouless (1931a) and Langdan (1953) were the reverse of those reported by Sheehand and Lichte. Langdan (1953) investigated the presumed changes in constancy over the arc of inclination under conditions which make it a reasonable assumption that the shape undergoing tilt continues to be perceived as physically unchanged. The comparison shapes were ellipses in the frontal-parallel plane representing projections of a circle. Observers compared these to a standard circular shape which was continually oscillating through an arc of 90 degrees. The subject then indicated when the oscillating circle and the ellipse appeared most similar in shape. The results showed an extremely high constancy toward the line of regard falling to a low point around 60-50 degrees, rising slightly thereafter and then declining once more as the frontal-parallel plane was approached (Langdan, 1953). Further conflicting results were submitted by other investigators (Moore, 1938).

An interesting note by Thouless (1972) questioned whether perceptual constancy or perceptual compromise has actually been studied by most investigators. Thouless explained that during a typical experiment in perceptual constancy, an unchanging physical object is observed under changing stimulus conditions (variation of inclination, distance or

illumination). The phenomenal character is then found to change less than the stimulus character. To this result, the experimenter attributes shape constancy. Thouless claimed that the opposite experimental situation, in which the phenomenon changes although the stimulus remains the same while the "real" character of the object changes has been relatively neglected.

An example of a typical experimental situation of this kind in shape perception may be provided by sitting a subject immediately under a projector which throws a circular image onto a screen. This image appears circular when the screen is at right angles to the line of vision.

Although the retinal image obviously remains circular, if the screen is rotated about 45 degrees around a vertical axis, all subjects report the apparent shape of the projected image as an elliptical image on the screen, but a circular retinal image (Thouless, 1972). Therefore, such observations suggest the inadequacy of a description of such perceptual effects in terms of a tendency for phenomena.

An early study on shape discrimination was performed by Arnoult (1954). He studied the relationship between angular orientation of stimuli and shape discrimination. The stimuli used were two-dimensional nonsense shapes which could be rotated in their own plane to any of eight positions. The accuracy and latency of shape discrimination were studied as a function of angular separation. The various combinations of positions of the two shapes in each pair resulted in 13 different amounts of angular separation. Pairs of two-dimensional nonsense figures were presented tachestoscopically to subjects. The subjects were then required to judge them for identity of shape. Errors ranging from 20 to 40 percent were observed for two shapes which were identical as the angular separation increased. For the

trials in which the two shapes were actually different, errors showed an initial decrease, followed by an increase and a final drop at 180 degrees. Hence, a significant interaction was found between angular separation and the variable of shape. Generalizations from these results, however, were later found to be limited.

Shape and Pattern Recognition

Attneave and Arnoult (1956) conducted a quantitative study of shape and pattern perception. The authors began by pointing out three problems in pattern recognition. The first is fact that shape is a multidimensional variable, though it is often carelessly referred to as a dimension along with brightness, hue, area and the like. Secondly, the complexity of the shape affects the number of dimensions necessary to describe a shape. Finally, even if the number of necessary dimensions in a given case are known, the choice of particular descriptive terms still remains a problem. Attneave and Arnoult expressed the need for an adequate psychophysical framework for these studies in which it is necessary to manipulate shape or pattern as an independent variable. The authors also developed methods for constructing nonsense shapes and patterns. Common to both shapes and patterns was the fact that the particular characteristics of each figure were randomly determined.

A Russian study on visual pattern recognition in man and animals was done by Glezer, Leushina, Nevskaya, and Prazdnikova (1973). Three types of recognition performance were revealed through the investigation of the processes involved in the recognition of visual pattern in man. These

included the fact that spatial properties such as line orientation are independent of the number of patterns presented for recognition. Also, the time characteristics of recognition of object pictures and geometric figures do depend upon the entire set of items expected by the observer in a particular situation. Finally, they found that after training with a limited set of patterns, the recognition type undergoes changes. That is, for each of the patterns, one complex feature is formed to make up a standard.

The three recognition types (described above) found in man were also used in conditioning experiments in animals (Glezer et al., 1973). Surgical cuts made within the visual cortex did not impair the organism's ability to recognize the line orientation. The line discrimination shifted to recognition through choice. This operation failed to affect the elaborated mechanism of figure recognition, though. The invariant recognition of the visual images in men is determined by the use of a visual system of separate channels which process the information concerning the shape of an object and its spatial characteristics, according to Glezer et al. (1973).

Invariance Hypotheses

The first formulation of an invariance hypothesis was made by Koffka (1935) in order to explain apparent exceptions to two principles which he believed to be basic to an understanding of perception. Koffka first observed that, under certain conditions, projective shapes which are discernibly different yield similar perceived shapes. This fact contradicted his principle that two proximal stimuli, if more than minimally different,

cannot produce exactly the same effect. Koffka's second principle states that proximal stimulus situations which are the same must produce the same perceptual effects. The fact that under certain conditions projective shapes which are identical yield different perceived shapes contradicts this second assumption. These apparent paradoxes may be resolved by noting that a perception produced by a given proximal stimulus pattern has at least two different aspects, shape and orientation.

Gibson (1951) elaborated on this point by stating that the problem of shape constancy is better formulated as the problem of seeing shape-at-a-slant. An impression of form is never obtained without some accompanying impression of the angle at which the surface lies, either frontal or inclined. In other words, perceiving a surface-form involves perceiving both the slant of the surface and the form of its edges. Therefore, what is invariant for a given retinal shape is not a given perception but a certain combination of apparent shape and apparent orientation. Thus, if the same retinal pattern gives rise to perceptions of two different shapes, the accompanying impression of slant will be such that the shape-slant relationship is invariant.

The difference in the percepts produced by two different proximal stimulus patterns is due to a shape-slant combination and not necessarily just the shape or slant alone. Therefore, if two different retinal patterns give rise to the perception of the same shape, it will be a shape perceived at different degrees of slant (Epstein and Park, 1963). Conversely, what is invariant for a given retinal shape is not a given shape perception but a certain combination of apparent shape and orientation.

Beck and Gibson (1955) assimilated these ideas into a shape-slant invariance hypothesis. According to Beck and Gibson, a retinal projection of a given form determines a unique relation of apparent shape to apparent slant. This formula asserts first that a retinal form determines a family of possible apparent shapes, and secondly that each of these apparent shapes is linked with a corresponding apparent slant. The invariance hypothesis implies (Epstein and Park, 1963) that for a retinal projection of a given form, a reduction in the accuracy of perceived slant will be accompanied by a corresponding reduction in the accuracy of perceived shape. If the observer makes an error in his perception of slant, then there should be accompanied deviations of phenomenal shape from the objective shape.

The relationship between slant and shape perception has been studied by a number of theorists. Beck and Gibson (1955) conducted one of the earliest investigations when they looked at the relation of apparent shape with apparent slant in the perception of objects. In their first experiment, Beck and Gibson demonstrated that a reduced retinal shape without stimulation for the slant of the surface can induce a whole family of apparent shapes, and does not necessarily determine the perpendicular cross-sectional member of the family. During their second experiment, an illusory shape was induced by an illusory slant, again for a reduced retinal shape. The phenomenal slant of such an object was found to be that of the textured background surface, no matter what its physical slant was. These two experiments did not support Beck and Gibson's slant-shape invariance hypothesis. That is, for a given form, the accompanying retinal projection determines a unique relation of apparent shape to apparent slant (Beck and

Gibson, 1955). A third experiment showed that constancy of the perceptions of shape appeared when stimulation slant was provided. Beck and Gibson also concluded that this third phenomenon is not universal.

A year later, Smith (1956) looked at the gradients of outline convergence and distortion as stimuli for slant. His subjects viewed monocularly four film forms. They included a white circle and a white rectangle on black backgrounds, and a black circle and black rectangle on white backgrounds. All four forms were shown in the frontal plane. They were also slanted, at five different slants, about a vertical axis. Circles were perceived better than rectangles at slants greater than 30 degrees. Though consistent, differences in the opposite direction for the smaller angles were not statistically significant. Smith also found that black figures were perceived more accurately than white ones, but these differences again, were not statistically significant. Finally, a straight-line function of the visual angle was found for the curve of error in judging slant.

A study by Epstein, Bontragen and Park (1962) attempted to extend and clarify Beck and Gibson's (1955) findings. Four basic extensions were made. First of all, the slant-induction effect was investigated when the background was also slanted from the frontal parallel plane. Secondly, a more exact test of the invariance hypothesis was created by employing a technique which permitted continuous variation of the comparison stimulus. A more accurate measure of the apparent slant of the target was also obtained. Finally, the influence of three different instructional sets on the judgement of shape was investigated. The apparatus used was a rectangular light-tight tunnel, seven feet in length. During the experiment,

all subjects agreed that the judgements of slant made at the end of the experiment corresponded well with the apparent slant of the target during the earlier judgements of shape. The slant-induction effect was obtained not only when the background was vertical, but also when the background was slanted at 20 degrees. Therefore, this experiment confirmed and extended Beck and Gibson's (1955) findings concerning the slant-induction effect. The results also agreed with Beck and Gibson's (1955) invariance hypothesis, except for one noticeable difference. Epstein et al. found that their results showed less adherence to the invariance requirements than did the results of Beck and Gibson, for both monocular and binocular observations. The main reason for this disagreement may have resulted from the fact that different measurement techniques were employed in the two experiments. In the earlier study, Beck and Gibson forced their subjects to choose between one of two extreme alternatives (i.e. objective or projective). In this situation, the subjects would have probably selected the shape which was most like the one they had perceived even when they had recognized that the two stimuli were not identical. Epstein et al. (1962) used a continuously variable comparison which enabled subjects to make more sensitive discriminations, and thus eliminated the above problem.

Two experiments which had bearing on the shape-slant invariance hypothesis were conducted by Winnick and Rogoff (1965). The experiments were an attempt to draw a research parallel between size constancy and shape constancy. Apparent objective slant scales were determined for four rectangles, two trapezoids, a random shape, and an ellipse, in the first experiment. To provide data needed to assess the predictions of shape as

a function of slant, based on the shape-slant invariance view, the second experiment measured judgements of forms. The forms used in the second experiment were the same as those used in the first, and were also set at the same angles of inclination, in the same apparatus. Support for the shape-slant invariance hypothesis was obtained from the second experiment. The two trapezoids, equivalent to rectangles slanted in opposite directions, showed some contrast in shape judgements, though not of as great magnitude as for slant estimates. The results also confirmed the prediction that along with underestimations of slant found at smaller angles, matches should be close to projected shape, and that overestimations of slant at larger angles should result in settings progressively farther away from the projected shape. Winnick and Rogoff (1965) concluded that the shape-at-a-slant influence directs perception towards retinal shape, but on the other hand, awareness of the slant itself may cause a movement of the perception toward the real object.

Two more experiments were reported a year later by Winnick and Rosen (1966) on the implications of the shape-slant invariance hypothesis. In the first experiment, the stimulus was varied in both its width and its slant to achieve matches to four standard angles. In the second experiment, width and slant were varied to match four standard widths. In both experiments, an apparatus was used in which variations in the slant and in width settings were highly ambiguous and subject to instructional sets. A rectangle was used as the stimulus, throughout. Winnick and Rosen attempted to develop for shape constancy, a parallel to experiments in which apparent size has been studied as a stimulus for perceived distance. They were interested in determining whether retinal shape could be as

effectively employed for the estimation of both shape and slant. In the first experiment, the resultant width settings were found to be close to the projected widths of the obtained angles of slant. The projected widths of the obtained angles of slant did not differ from the obtained width settings in the second experiment. Close agreement between slant settings and width settings was shown in the two experiments. These results gave only limited support to the shape-slant invariance hypothesis since they were impaired by the narrow range of width and slant judgements involved (Winnick and Rosen, 1966).

Kaiser (1967) suggested that obtained changes in reported shape are primarily due to the effect of changes in perceived slant. His subjects described the shapes and slants of trapezoids by means of appropriate response mechanisms. The slant and shape judgements were made both monocularly and binocularly. The results showed that changes in reported shape varied as a function of changes in reported slant. Kaiser also found that when subjects had no prior binocular experience with the trapezoid, shape response errors varied as a function of slant response errors under monocular viewing. Again, the functions relating perceived shape to perceived slant were comparable to the function predicted by the Beck and Gibson (1955) shape-slant invariance hypothesis.

Kaiser (1967) also outlined three methodological improvements, which he used, that may have helped to obtain data in accord with the function predicted by the invariance hypothesis. First of all, the shape and slant response mechanisms were viewed under the same conditions as the standard stimulus. This was achieved by housing them both in a common

viewing box. Secondly, the subjects could report all aspects of their perception of the shape of the stimulus since the shape response mechanism had sufficient degrees of freedom. Finally, Kaiser insured that simultaneous shape and slant responses were approximated by allowing subjects to make adjustments to their initial responses until they were satisfied that both responses represented their perception of the shape and slant of the stimulus.

Motion and Shape Constancy

The perception of motion and subjective movement has been studied by a number of investigators. This area involves at least three separable but closely related problems, according to Gibson (1954). First of all, how do we see the motion of an object? Secondly, how do we see the stability of the environment? Finally, the problem of how we perceive ourselves as moving in a stable environment. The apparent expansion of the visual field has been noticed by nearly everybody who has driven in an automobile. The retinal motion reaches high magnitudes during rapid travel, and there is reason to believe that it is the important factor in the performance of landing an aircraft (Gibson, 1954). The flier is never confused by the impression that his runway is behaving like rubber. The question that arises is, why does the visual world not seem to expand, but instead appear rigid with the observer moving?

Gibson (1954) wrote that a promising hypothesis for research would be that any transformation of the total retinal image, as distinguished from a part image within it, tends to yield an experience of a movement of the

observer and the kind of movement experienced depends on the kind of transformation. A simple translation of the image may contribute to the experience of an eye movement, for example. In conclusion, Gibson (1954) claimed that the various motions of objects in a stable environment and the various movements of ourselves in that environment can both be visually perceived. If the effective stimulation is taken to be ordinal and relational, it falls into several mathematical classes, which may prove to be psychophysically correlated with modes of kinetic experience.

Gibson (1957) defined optical motion as a projection in two dimensions of a physical motion in three dimensions. This projection may be taken either as on the surface of a sphere centered at the eye or as a plane in front of the eye. The question for psychologists is, what kinds of optical motion occur which might serve as stimuli for perception? Gibson goes on to describe two possible answers to this question. First of all, one could divide the visual pattern into coordinate values. Then one could describe the optical motion by successive pairs of values. Similarly, motions of the elements could be described by direction and speed at successive moments of time. A second, non-analytical method of specifying optical motions is also possible. One could simply take the pattern as given and then use the operation defining a perspective transformation in geometry to describe a family of changes of pattern.

Gibson (1957) also described the six parameters of motion. The first two are translations, vertical and horizontal, in the plane. Vertical and horizontal foreshortening of the pattern may also occur. The pattern in the plane may also be enlarged or reduced, and finally rotated. Hay (1966)

added two more parameters in his study of optical motions in space perception. The discrepancy arose because Gibson (1957) was only looking for correspondents of six parameters of displacement, and neglected the two parameters of surface orientation at the start of the displacement. Of the six rigid phenomenal motions that Gibson (1957) described, three (rotation around the line of sight, transportation up or down, and transposition right of left) are induced by a stimulus which common sense would call motion (Gibson and Gibson, 1957). One would usually call a transposition along the line of sight by a stimulus, expansion or contraction. Furthermore, rotation around a horizontal or vertical axis is commonly referred to as a transformation.


Sidorsky (1958) examined absolute judgements of static perspective transformation. This study indicated that the underestimation of the slant of static perspective transformation of a regular pattern found by Gibson and Gibson (1957) is not a consequence of the lack of motion of the stimulus. Sidorsky concluded that if observers are informed that the differences in the configuration of a display are the result of a perspective transformation of a rigid surface, a series of line patterns never seen in motion can also carry a transformation sequence.

Fieandt and Gibson (1959) studied the sensitivity of the eye to two kinds of continuous transformations of a shallow-pattern. They included an experiment to compare the perception induced by two kinds of transformation sequences, with everything else kept as equal as possible, to determine whether corresponding impressions of rigid and of non-rigid motion would occur spontaneously. For an apparatus, Fieandt and Gibson used a

point-source shadow projector which was designed to produce an ongoing perspective transformation. They also modified the equipment so that it would also produce an ongoing transformation of the "rubbery" kind so that it could be shifted from the first to the second kind of transformation without any realization by the observer that the apparatus had been changed. Results from the experiment indicated that discrimination between motions by the observers was very efficient.


While a mathematical analysis of rigid motions is quite straightforward (Braunstein, 1966), the generation of visual displays in conformity with such an analysis was not practical until Green (1961) introduced a new concept in preparing motion pictures. These motion pictures were made for stimulus displays by plotting each individual frame from mathematically generated coordinates. A digital computer, digital to-analog converter, cathode-ray-tube plotting device and motion picture camera were used in this technique.

One general finding from Braunstein's (1966) experiment was that no texture generated as three-dimensional was judged to be two-dimensional by significantly more than fifty percent of the subjects. Also, the textures which were judged as two-dimensional by most subjects had been generated as two-dimensional and underwent transformations yielding a clear distinction between two and three-dimensional textures. These findings support Gibson's (1951) concept that three-dimensional perception is primary and two-dimensional perception requires the addition of a special thought process. Support is also provided for the postulate that the human observer is sensitive to transformations of patterns of light stimulation.



to the retina, and can perceive motion and depth on the basis of such transformations. Braunstein (1966) explained that these transformations may contain information leading to the perception of three dimensions.

Massaro (1973) studied shape constancy through the perception of rotated shapes. His subjects made objective shape judgements of rotated circular objects. This provided a process analysis of shape constancy. Massaro presented evidence suggesting that the critical independent variable in the shape judgement task is not the angle of rotation of the figure, but rather the distance between the nearest and farthest points of the test figures. Employing test figures which had narrower horizontal axes decreased the effect of angle of rotation. Massaro concluded that the perceptual encoding model provides the best description of the time it takes to determine the shape of a figure when it is rotated in depth.



Charness and Bregman (1973) conducted three experiments to study transformations in the recognition of visual forms. The experiments were based on a view of pattern recognition as a constructive process in which the input is given a structural description in terms of some known and recognizable concepts. Their subjects were required to learn to recognize four flexible plastic shapes. These shapes were photographed on different backgrounds from different angles. Amount of training, type of training instructions, presence or absence of context (background which gave cues to transformation) in training and in testing, and order of testing were the variables which Charness and Bregman (1973) investigated. They found that when the subjects were informed verbally that the variations in a set of photographs had been produced by projective transformations, the

ability to extract the real-shape of the object was increased. Results indicated that subjects can learn an ideal shape with very few trials if that ideal shape is isolated for them or if context can be utilized to account for transformations of the shape. Conditions which favour the extraction of real-shape parameters permit better recognition of projective distortions. Finally, Charness and Bregman found that when conditions do not favour the extraction of real-form parameters at the time of training, performance is increased by the presence of visual context at the time of testing. The importance of a context rich environment for learning and recognition of visual patterns was illustrated by this study.

Higher-Order Invariants

According to Gibson (1959), for the observer in motion, the relevant variables are the higher-order invariants and stimulus gradients that confront him. This statement refers to the question of how we recognize familiar shapes while we are in motion. For example, why do we say that a window is rectangular, as we pass by it, even though the only time in which the corresponding retinal image is actually rectangular, is that split-second in which we are directly in front of the window? The rest of the time, the retinal image is a trapezoid which gradually approximates a rectangle, as we walk up to the window, then becomes distorted again as we pass by.

Gibson (1966) regards the different senses as active seeking mechanisms for looking, listening, touching, and so on. These perceptual systems are

not treated by Gibson as mere producers of visual, auditory, tactual, or of other sensations. This means that the important question, which should be asked in perception, is how we are able to obtain constant perceptions that we need for effective action and avoidance of physical harm in our everyday lives?

Gibson (1966) also emphasized the importance of regarding the different perceptual systems not only as active but also interrelated. He clearly supports the view that perception of reality is not simply something assembled or computed by the brain from ever-varying sensations. For example, Gibson points out that visual perception is frequently a seeking of information by head and eye movements coordinated in the so-called eye-head system. Information about the world is searched for and extracted by this outreaching mechanism.

Continuous stable information (Gibson, 1966) that makes adaptive living possible, is provided by our active interrelated perceptual systems. Therefore, observers are forever searching, through their perceptual systems, for the invariant cues in the environment which provide them with a constant perception.

Gibson (1966) claimed that certain higher-order variables of stimulus energy do not change. That is, besides the changes in stimuli from place to place and from time to time, there exist ratios and proportions, for example, which remain constant. The invariance of the stimulus energy at the receptors of an organism correspond to the permanent properties of the environment.

Observers in motion also receive invariant perceptions despite varying sensations. They perceive a constant object by vision despite the distorted retinal image received. The hypothesis states that constant perception depends on the ability of the individual to detect the invariants, and pay no attention to the flux of changing sensations (Gibson, 1966). Therefore, the transformations of a pattern or shape in motion must specify the invariants of such a stimulation.

If one takes this Gibsonian stand, the puzzle of constant perception despite varying sensations disappears and a new question arises (Gibson, 1966). We now ask how the invariant information is extracted from a changing environment. Therefore, a true perceptual system hunts for a state that we may call clarity or constancy in our everyday surroundings.

Hockberg (1974) commented that Gibson's most powerful concept, and the one that unites and underlies many of his interests and proposals, is his idea that there exist higher-order variables of stimulation to which the properties of the objects and events that we perceive are the direct and immediate response. Gibson (1959) has stated that most of the properties of the perceived world are evoked directly by the variables of stimulation that the sensory system receives from the normal ecology.

Previous Mathematical Analyses

A number of mathematical analyses of visual perception have been proposed in the past. One of the first, and one which formed a basis for many later theories, may be found in a book on binocular vision by Luneburg (1947).

In it, the geometry of visual space is demonstrated to be non-Euclidean. Luneburg showed that visual space must be one of the Riemannian spaces of constant positive, negative or zero curvature.

Following Luneburg's approach, Gilinsky (1951) studied mathematically, perceived size and distance in visual space. The purpose of her study was to develop a quantitative formulation of visual space perception, expressing functions relating visually perceived size and distance to the true size. Her formulae were rigorously derived from the basic metric of visual space which had been established by Luneburg (1947).

A follow-up paper by Fry (1952), however, indicated that Gilinsky (1951) had not actually followed all of Luneburg's basic assumptions. Instead, Gilinsky, in order to derive her equations, had assumed that visual space was in fact Euclidean. Using this approach though, she did produce an equation which fitted a reasonable amount of data. The formulae were also in agreement with the logically deduced laws of complete size constancy for objects close at hand, and of the retinal images for remote objects (Fry, 1952).

Following the Luneburg (1947) approach more closely, Blank (1958) concluded that there is no advantage in postulating an Euclidean metric. According to Blank, the visual metric cannot be equated to the physical metric. For example, physical and visual length orderings differ and visual straightness is not the same as physical straightness (Blank, 1958). In his paper, Blank proposed basic axioms which formed a foundation of metric geometry in relation to space perception.

A paper by Smith (1959) showed that the negative curvature of visual space can be derived entirely by mathematical arguments, given certain initial assumptions. Following Blank's (1958) approach, the probability dispersion of points in the visual field was examined. The stimuli considered were isolated point-sources in otherwise dark surroundings. This study led to an expression for the uncertainty of the scale of measurement. When this scale of uncertainty is eliminated by a mathematical transformation, a Riemannian space of constant negative curvature results (Smith, 1959).

In a study on the parallax and perspective during aircraft landing by Gibson, Olum and Rosenblatt (1955), a mathematical analysis of motion was presented. This was done in terms of the optical flow-pattern reflected from a surface to an eye. Gibson et al. found that the variables of this flow-pattern are specific not only to the depth of the surface, but also to the movement of an observer. They applied this finding to the problem of perceiving the ground and a pilot's relation to it during aircraft landings.

An extension to Gibson's (1955) analysis of optical motions and space perception was proposed by Hay (1966). The mathematical analysis in Hay's paper was organized around the question of how many different object displacements can produce one and the same optical motion. The study showed that displacements of objects, so long as they are flat, are mapped into projective transformations of the optical array. That is, a form of a flat surface produced the same projective transformation in the optical array, as long as it undergoes a specific displacement from a specific position (Hay, 1966).

Tonge (1976) attempted to construct a general mathematical theory of perception which would be in agreement with available experimental results. Using a mathematical model, Tonge theorized that the transmission of sensory signals from neurons at a given level to those at a subsequent level may be viewed as a transformation from one quantized space to another. This transformation was assumed to be subject to the condition that certain features of the environment which are essential to the production of a response, remain invariant.

In a recent paper by Hadani, Ishai and Gur (1980), a mathematical model of monocular space perception was presented. One of the basic assumptions used in the analysis was that the head of the observer was fixed and the visual field was static. According to Hadani et al. (1980) three-dimensional spatial coordinates of viewed objects may be reconstructed using a method of infinitesimal transformations. These transformations follow the detection of retinal luminance changes due to involuntary eye movements and the analysis of the angular velocity of each image point.

The Present Analysis

This present paper will attempt to analyse mathematically the transformation-invariant cues involved in the recognition of shapes. This study will follow Gibson's (1959) approach of looking for higher-order variables of optical stimulation which correspond closely with the environment. In particular, an examination of the visual cues which allow people to perceive objects as stable and invariant, from differing angles of regard, will be made. For instance, this study will cover the kind of transformation-invariant cues present that observers receive visually as they pass a window.

In this example, the physical retinal image would approximate a trapezoid even though a rectangular object is usually perceived and reported.

This paper will be restricted to the study of how two-dimensional objects are perceived. These objects, however, will be at times slanted away from the observer. There are basically three different stimulus situations which are generally used in the study of dynamic slant perception (Braunstein, 1976). The first is an actual rotation of a surface, usually about a vertical or horizontal axis. The second class of transformations which may be studied are surfaces being translated across an observer's field of view. Finally, slanted objects may be moved towards or away from an observer. This present study will only be concerned with a subset of the second type of stimulus situation. In particular, two-dimensional objects which are translated horizontally in the observer's frontal plane will be examined.

In order to properly study the transformation-invariant cues involved in the translation of an object in an observer's frontal plane, the observer will be assumed to be fixed. Then the visual cues present will, first while the object is also stationary and then in horizontal motion, be considered. Comparing these two cases mathematically, a static then moving object, should expose any transformation-invariant cues which may be present.

To fully investigate all transformation-invariant cues, arbitrary two-dimensional objects will be analyzed mathematically. After this is completed, the results will be extended to the familiar shapes of rectangles and circles. This will not only show how the formulae may be applied,

but will also point out any transformation-invariant cues specific to these common two-dimensional objects. Then the question of how people perceive rectangles and circles as such, and not as trapezoids and ellipses, respectively, will be better understood.

CHAPTER II

PLAN OF ANALYSIS

The analysis will be organized around the question: What are the transformation-invariant cues which facilitate the recognition of two-dimensional objects being translated in an observer's frontal plane? Hence, the experimental situation in which subjects view two-dimensional objects passing in their frontal plane will be analyzed mathematically. This will roughly correspond to a number of real life situations, including, for example, a person walking by and observing a window. For analysis and experimental reasons though, the subject will be kept stationary and the object put in motion. The same transformation cues, of course, will be present.

The Experimental Setting

The experimental situations which will be analyzed are illustrated in figures 1, 2 and 3. The observer's position is identified by S and is assumed to be stationary. The orthogonal axes X, Y and Z have been labeled following Braunstein's (1976) approach. That is, the Z (depth) axis refers to the observer's line of sight, when the subject looks straight ahead. The X (horizontal) axis is perpendicular to the line of sight and parallel

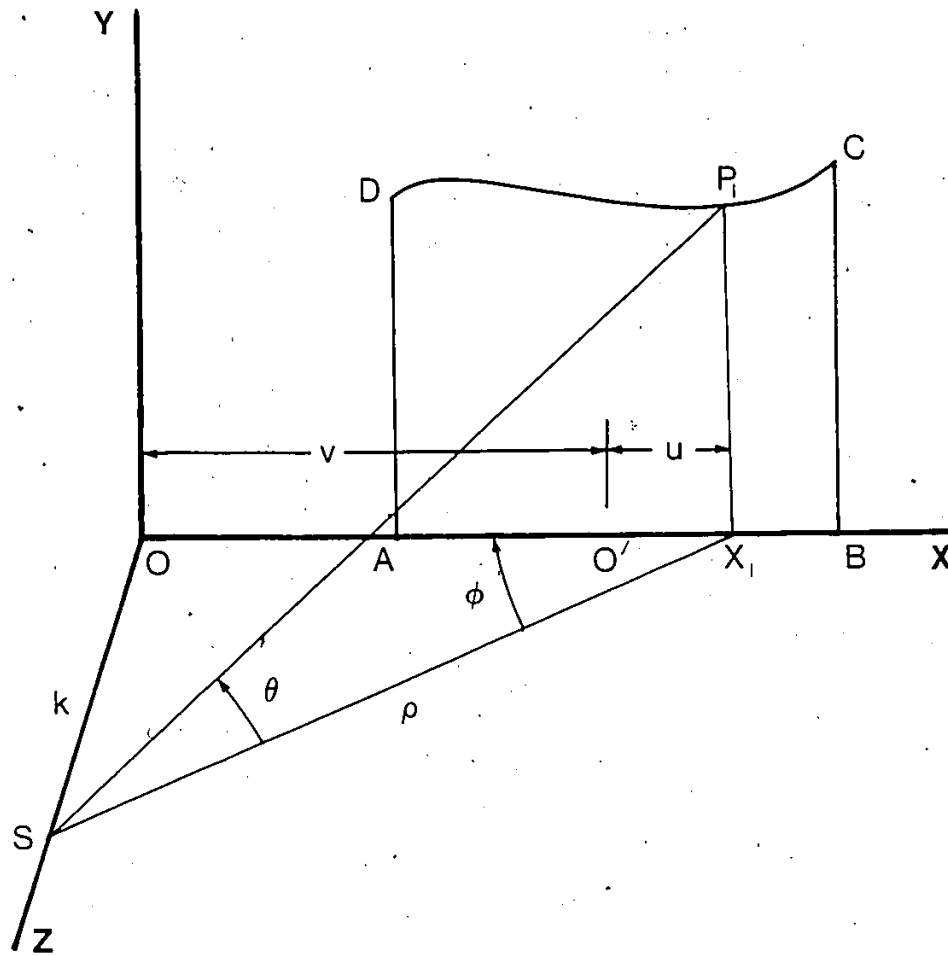


Figure 1. Geometric representation of an arbitrary two-dimensional object displayed in the frontal plane.

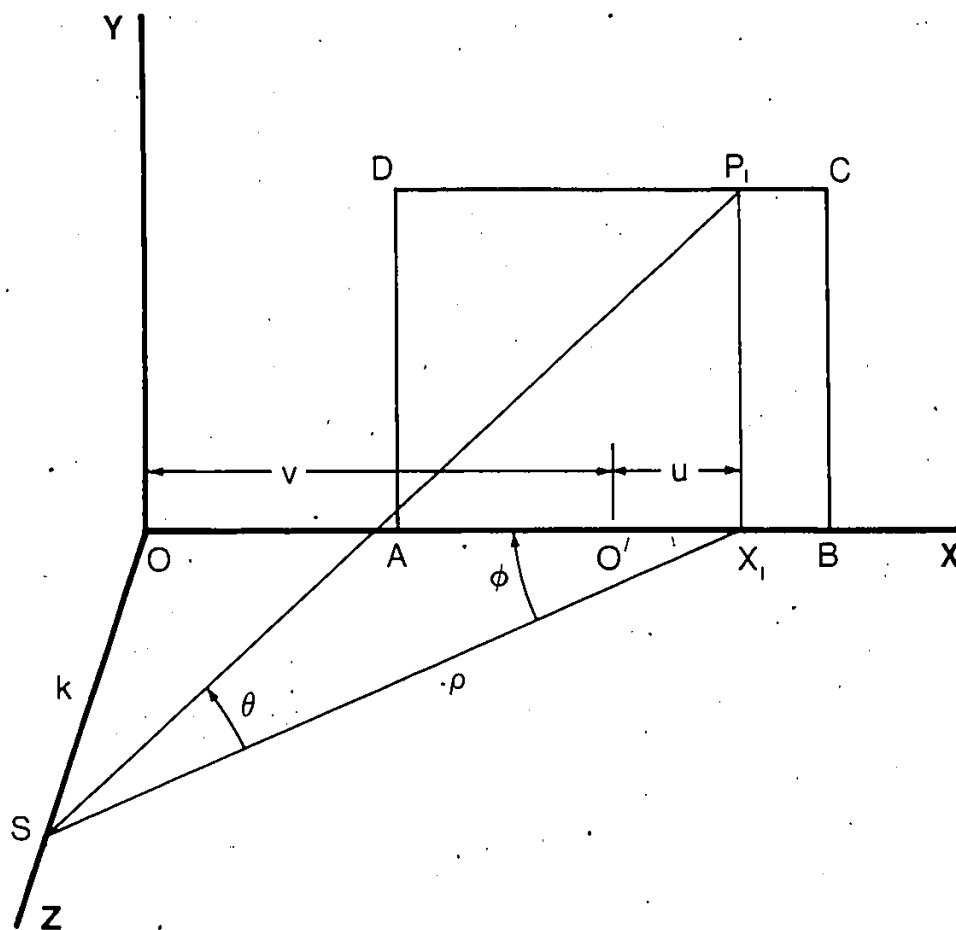


Figure 2. Geometric representation of an arbitrary rectangle displayed in the frontal plane.

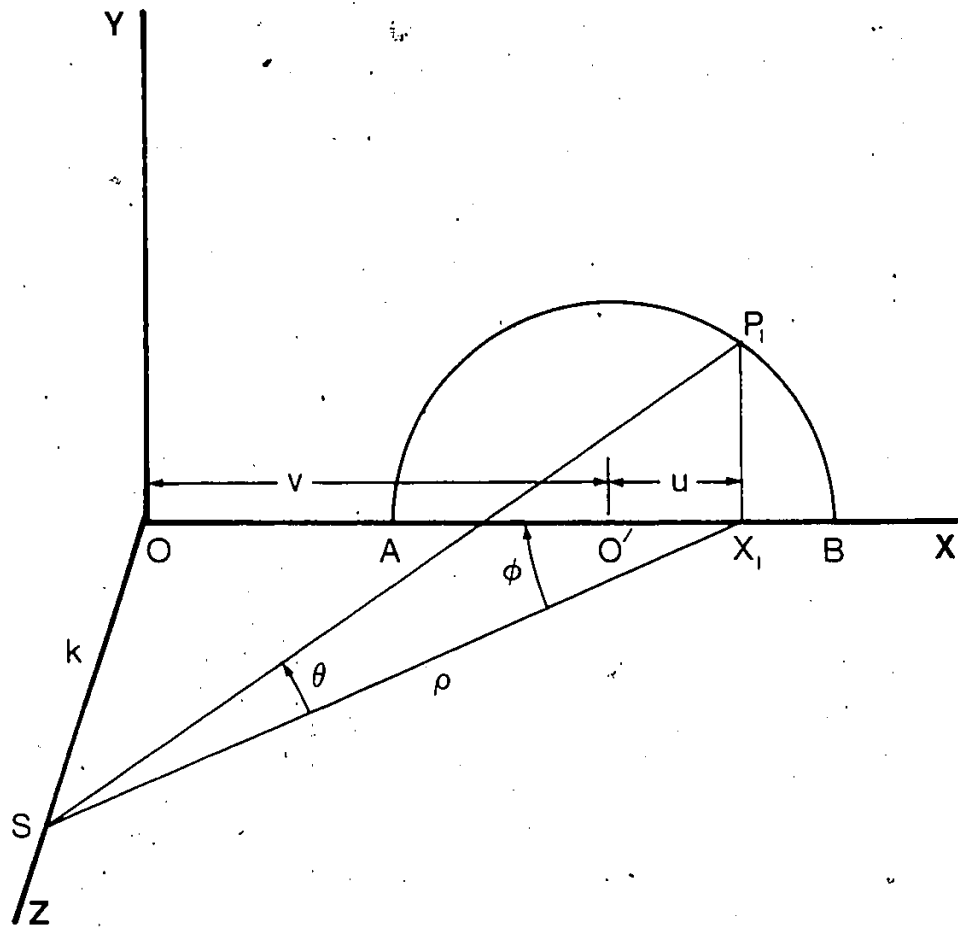


Figure 3. Geometric representation of an arbitrary circle displayed in the frontal plane.

to the line connecting the subject's two eyes. Finally, the Y (vertical) axis is defined to be perpendicular to both the Z and X axes. In this way, the three-dimensional space present in an actual experiment will be defined.

To facilitate easier formula manipulation, the lengths of the line segments will be assigned single letter variable names. In particular, the length of the line SO will be denoted by k . In this present study, k will be regarded as any arbitrary constant value since it will be assumed that the observer's position will remain fixed.

An analysis of arbitrary objects will be conducted first to obtain a general finding for all two-dimensional forms which may be displayed horizontally in an observer's frontal plane. Therefore, an arbitrary object will be translated or fixed in some position along the X axis (see figure 1). To simplify the analysis, only the top half of the object will be considered. This will not restrict or hinder any conclusions made from this study as generalization to vision of the whole object can later be made quite easily.

The upper half of the object, the part which will be studied, will have the points A, B, C and D as corner edges, as shown in figures 1, 2 and 3.

These may not be the only corner edges on the object, as in the case of a hexagon. Points A and B will, however, indicate uniquely where the edge of the object passes through the X axis. Points C and D will correspond to the end-points of the line defining the top of the object. In the case of a circle, as seen in figure 3, points A and D, and B and C coincide and lie on the end-points of a semi-circle. For computational ease, the lines

AD and CB will be assumed to be vertical. Also, no spaces or holes within the ABCD outline will be considered. However, generalizations to more irregular shapes may be easily made, given the basic analysis results.

In figure 1, the point O' has been designated as the center of the object. In the case of a circle (see figure 3), O' would, of course, refer to the center of the circle wherever it may lie on the X axis. If a more irregular object was being studied, the center point might not be as obvious. In this latter case, any agreed upon point within the object would suffice as long as it stayed fixed within the object.

The length of the line OO' (see figure 1) will be denoted by the value v , which will correspond to the distance from the center of the object to the point O . Naturally, point O , the origin of the three-dimensional space, denotes the closest point on the XY plane to subject. When the object is centered directly in front of the observer, the point O' will coincide, of course, with the origin O . The value v will be fixed at some arbitrary amount or varied depending on whether the object being studied is assumed to be static or in a moving state, respectively.

The variable u will be used to represent the difference $OX_1 - OO'$ (see figure 1). The point X_1 indicates the position at which the observer's line of sight intersects the X axis. This position is, of course, not fixed. Hence, the absolute value of u will vary from zero to the width of the object, the length of the line AB, as the observer scans the shape.

The length of the line OX_1 will then equal $u + v$. Also, the height of the arbitrary object will be defined in the XY plane by $y = f(u)$.

The angular height, denoted by θ in figures 1, 2 and 3, will be used in an equation to express the physical cues present that an observer receives. This angle depends not only on the size of the object but also on the distance and orientation of the object in relationship to the subject's position S. Though θ defines the angle subtending a single line segment only, for example, $X_1 P_1$ in figure 1, the changes in the angular height over the entire length of the object, from AD to BC, will define the shape exactly. From a defining equation involving θ , not only the shape, but also the orientation of the object to the observer may be derived.

The second angle to be considered will be the slant angle, denoted by ϕ in figures 1, 2 and 3. This value will be used to express the angle between the line of sight and the XY plane. In other words, the difference in direction of the two line segments SX_1 and $O'X_1$. Hence ϕ will indicate the degree to which the object is slanted away from the observer.

The Basic Analytical Approach

To fully understand the influence of motion in the recognition of objects, the study will proceed in three main steps. First, a study of the cues obtained from static objects will be made. In this case, the point O' (see figure 1) will be fixed at some arbitrary distance from the origin O. A study of the defining equation of the object, in terms of the angular height will then be made. This will indicate the information observers receive as they regard a still two-dimensional shape.

Secondly, an examination of the transformation cues which observers receive

as an object moves horizontally across their frontal plane will be made. To achieve this, v will be varied. That is, point O' will be translated along the X axis towards or away from point O . Changes in the defining equation, due to changes in the object's orientation with regard to the observer, will be examined.

Finally, the results of steps one and two will be compared. That is, the equations obtained by studying the static and moving states will be compared to discover any possible similarities or differences in the type of information they yield. In particular, a search for any added information that an object in motion gives when compared to a static one will be done.

Two Specific Cases

After an arbitrary object has been investigated in both the static and moving states, two familiar forms will be studied using the same approach. This will be done for basically two reasons. First to demonstrate how the derived general formulae may be employed, and secondly to point out the transformation-invariant cues specific to these common shapes.

A rectangle will be considered first (see figure 2), and will have a defining equation of $y = c$, where c is some constant. The width of the rectangle will also be set at some constant value w , which will correspond to the length of the line segment AB in figure 1.

The circle will be the second common object which will be studied. The shape (see figure 3) will be defined by the equation $y = \sqrt{r^2 - u^2}$, since

only the upper half (the part above the X axis) will be examined. The value r , the radius of the circle, will equal the line segment AO' , or equivalently $O'B$ in figure 3.

An arbitrary trapezoid and ellipse will be examined following the initial analysis of the rectangle and circle respectively. This will indicate how the first two regular objects correspond to more irregular shapes which approximate them. In other words, this part of the study will analyse how observers are able to discriminate between rectangles and trapezoids, and circles and ellipses, which are presented at a slant. The arbitrary trapezoid and ellipse will be defined by $y = au + b$ and $y = m \sqrt{n^2 - u^2}$, respectively. The width of the trapezoid will be set at some arbitrary value, w , as in the case of the rectangle.

CHAPTER III

THE CATEGORIZATION OF VISUAL INFORMATION OBTAINED FROM TWO-DIMENSIONAL OBJECTS

Since only two-dimensional objects which are presented in an observer's frontal plane, or equivalently the XY plane as shown in figures 1, 2 and 3, will be considered, the angular height θ will always be defined as

$$\theta = \tan^{-1}(y/\rho) \quad (1)$$

$$\text{where } \rho = \sqrt{(u + v)^2 + k^2} \quad (2)$$

Note that the angular height is a function of the variables u , v and y and the constant k . These four values together, are necessary and sufficient to define any two-dimensional objects and their orientation with regard to an observer, given the basic constraints of the present experimental situation.

Stationary Objects

The perception of a stationary arbitrary two-dimensional object will now be considered. In this case, the value v , corresponding to the length of

the line segment $00'$, will be assumed to be fixed. Given the formula for the angular height (see equation 1), the change in shape, or more specifically, height, with regard to time t , may be expressed as follows

$$\frac{d\theta}{dt} = \frac{\rho \frac{dy}{du} \frac{du}{dt} - y \cos \phi \frac{du}{dt}}{\rho^2 + y^2} \quad (3)$$

$$\text{where } \cos \phi = \frac{u + v}{\rho} = \frac{d\rho}{du}$$

Equation 3 may be easily decomposed by letting

$$\text{component I} = \frac{\rho}{\rho^2 + y^2} \frac{dy}{du} \frac{du}{dt} \quad (4)$$

and

$$\text{component II} = \frac{y \cos \phi}{\rho^2 + y^2} \frac{du}{dt} \quad (5)$$

Then clearly,

$$\frac{d\theta}{dt} = \text{component I} - \text{component II} \quad (6)$$

The change in u with regard to t (i.e. du/dt) will be referred to as the scanning velocity.

Moving Objects

Before a further analysis of these components, given in equations 4 and 5 is done, a study of an arbitrary moving two-dimensional object will be

made. In this second case, v will not be fixed. Note that the two variables u and v will be assumed to be independent of one another. In other words, the observer's scanning of the object will not be dependent on the position of the object. The perceptions received from this scan will, however, depend on the location of the shape.

In order to take into account all the changes to angular height with regard to the independent variables u and v , the total derivative with respect to time to may be derived as follows

$$\frac{d\theta}{dt} = \frac{\rho \frac{dy}{du} \frac{du}{dt} - y \cos \phi \frac{du}{dt} - y \cos \phi \frac{dv}{dt}}{\rho^2 + y^2} \quad (7)$$

$$\text{where } \cos \phi = \frac{u + v}{\rho} = \frac{d\rho}{du}$$

The derivative dv/dt will be referred to as the translation velocity. As in the static case, equation 7 may easily be decomposed by letting

$$\text{component III} = \frac{\rho}{\rho^2 + y^2} \cdot \frac{dy}{du} \frac{du}{dt} \quad (8)$$

and

$$\text{component IV} = \frac{y \cos \phi}{\rho^2 + y^2} \left(\frac{du}{dt} + \frac{dv}{dt} \right) \quad (9)$$

then clearly,

$$\frac{d\theta}{dt} = \text{component III} - \text{component IV} \quad (10)$$

A Comparison of the Static and Moving States

To understand the relationship between the static and moving cases, a comparison of the components which comprise the angular height derivative formulae (see equations 6 and 10) will now be made. To begin with, note that the components I and III, given in equations 4 and 8, are essentially equal, except that in the latter case, the value v is assumed to be variable instead of constant as in the first expression. The components I and III contain the part of the change in the angular height which is due to a change in y (i.e. dy/du). This change in y , or the height of the object, with regard to u , defines the shape of the object when it is totally scanned. Note that this change in shape is independent of the position of the object in the XY plane. Therefore, components I and III contain what may be termed true shape (i.e. dy/du and y) and distance (i.e. ρ) information.

The difference between the static and moving states may now be assumed to result from a discrepancy between components II and IV, given in equations 5 and 9. Both components contain true shape, distance, and slant information given by the values y , ρ and $\cos \phi$ respectively. These factors together will be referred to as distortion due to slant (i.e. $(y \cos \phi) / (\rho^2 + y^2)$). The difference between components II and IV arises from a discrepancy in the velocity terms. In the static case, the distortion due to slant is multiplied by only the scanning velocity. On the other hand, in the second case when the object is not fixed, the distortion due to slant is multiplied by both the scanning and translation velocities.

Rectangles and Circles

The specific case of an arbitrary rectangle which may be translated horizontally across a subject's frontal plane, as illustrated in figure 2, will be analyzed now. As indicated earlier, the equation of the line DC in figure 2 will be given by $y = c$, where c is some arbitrary constant. When the rectangle is held stationary by keeping v fixed, formula 6 yields

$$\frac{d\theta}{dt} = - \frac{c \cos \phi}{\rho^2 + c^2} \frac{du}{dt} \quad (11)$$

In this case, component I (see equation 4) which is evident in the general arbitrary object case, reduces to zero. Thus the change in angular height is only affected by the distortion due to slant.

If the value v is allowed to vary, the total derivative of the angular height may be derived using equation 10 as

$$\frac{d\theta}{dt} = - \frac{c \cos \phi}{\rho^2 + c^2} \left(\frac{du}{dt} + \frac{dv}{dt} \right) \quad (12)$$

Again, as in the case of the static rectangle, the change in the angular height is affected only by the distortion due to slant, but now times both the scanning and translation velocities.

Equations similar to those in formulae 11 and 12 may be derived for an arbitrary trapezoid which is defined by $y = \lambda u + b$. First, for the static case in which v is held constant,

$$\frac{d\theta}{dt} = \frac{a\rho}{\rho^2 + (au + b)^2} \frac{du}{dt} - \frac{(au + b) \cos \phi}{\rho^2 + (au + b)^2} \frac{du}{dt} \quad (13)$$

Secondly, when v is not fixed, the derivative of the angular height for an arbitrary trapezoid with respect to time t is given by

$$\frac{d\theta}{dt} = \frac{a\rho}{\rho^2 + (au + b)^2} \frac{du}{dt} - \frac{(au + b) \cos \phi}{\rho^2 + (au + b)^2} \left(\frac{du}{dt} + \frac{dv}{dt} \right) \quad (14)$$

Note that in the case of the trapezoid, components I and III do not reduce to zero as in equations 11 and 12 which were derived for an arbitrary rectangle.

Using the same approach as in the analysis of the rectangle, a circle will now be studied. In this case, an arbitrary semi-circle connecting AB in figure 3 will be defined by $y = \sqrt{r^2 - u^2}$, where r refers to the radius of the circle. Considering first the static situation and using equation 6

$$\frac{d\theta}{dt} = -\frac{\rho u (r^2 - u^2)^{-1/2}}{\rho^2 + r^2 - u^2} \frac{du}{dt} - \frac{\sqrt{r^2 - u^2} \cos \phi}{\rho^2 + r^2 - u^2} \frac{du}{dt} \quad (15)$$

is found. Now assuming that the object is free to move along the X axis, the following may be derived using equation 10.

$$\frac{d\theta}{dt} = -\frac{\rho u (r^2 - u^2)^{-1/2}}{\rho^2 + r^2 - u^2} \frac{du}{dt} - \frac{\sqrt{r^2 - u^2} \cos \phi}{\rho^2 + r^2 - u^2} \left(\frac{du}{dt} + \frac{dv}{dt} \right) \quad (16)$$

To accompany the analysis of the circle, an arbitrary ellipse will now be examined. Equations corresponding to formulae 15 and 16 will be derived assuming the object is defined by $y = m \sqrt{n^2 - u^2}$, where m and n are

constants. First when the ellipse is assumed to be stationary, and therefore v is fixed, from equation 6, the following may be derived.

$$\frac{d\theta}{dt} = -\frac{m \rho u (n^2 - u^2)^{-\frac{1}{2}}}{\rho^2 + m^2 (n^2 - u^2)} \frac{du}{dt} - \frac{m \sqrt{n^2 - u^2} \cos \phi}{\rho^2 + m^2 (n^2 - u^2)} \frac{du}{dt} \quad (17)$$

Then when v is allowed to vary, using equation 10, the derivative of the angular height with respect to time t is given by

$$\frac{d\theta}{dt} = -\frac{m \rho u (n^2 - u^2)^{-\frac{1}{2}}}{\rho^2 + m^2 (n^2 - u^2)} \frac{du}{dt} - \frac{m \sqrt{n^2 - u^2} \cos \phi}{\rho^2 + m^2 (n^2 - u^2)} \left(\frac{du}{dt} + \frac{dv}{dt} \right) \quad (18)$$

CHAPTER IV

DISCUSSION

The analysis examined the transformation cues involved in the recognition of two-dimensional objects. These objects were assumed to be presented in the observer's frontal plane. Static and moving cases involving an arbitrary object were analyzed first and then compared to expose the influence motion has on the perception of shapes. These results were then applied to arbitrary rectangles, trapezoids, circles and ellipses.

The Angular Height

The angular height, defined in equation 1, was used in this study to express the physical impressions that observers receive while viewing two-dimensional objects in their frontal plane. In a study on the recognition of lunar craters, Wilde, Siegel and Williams (1967) used the apparent image of an object instead of the angular height. In their case, an ellipse was used to simulate a situation in which a subject observes a lunar crater.

There are at least two disadvantages associated with the use of an apparent image, as opposed to the angular height. An example of the first may be found in the study by Wilde et al. (1967) in which the apparent image of a

circle was taken to be elliptic, even though it should have only approximated an elliptical form. In this present study, the exact physical perceptions were examined, both in terms of shape and the object's orientation with regard to the observer. The second disadvantage arises from the restriction that only specific objects may be studied when the apparent image approach is utilized. As in the paper by Wilde et al. (1967), where the eccentricity of the apparent image was examined, every object studied would need an accompanying physical measure which would be related to changes in shape. When the angular height is used instead, an arbitrary form may be studied, as was demonstrated in this paper. Overall, the angular height affords an investigator more flexibility and precision when examining two-dimensional objects.

Visual Information Components

This present paper first investigated the transformation cues received from stationary objects. In order to derive general formulae, an arbitrary shape was assumed. An equation (see formula 3) expressing the changes in the angular height, with respect to time, as a fixed observer scanned the object, was decomposed into two parts. The first component (see equation 4) may be thought of as the true shape and distance factor.

To better illustrate the notion of a true shape and distance component, an example of a situation in which the value ρ is held constant is now given. In this case, only the first component in equation 6 exists as the second reduces to zero, since $d\rho/du = 0$. That is, $d\theta/dt = \text{component I.}$ Hence, there should be no distortion due to slant present, and the true shape and distance should be easily perceived.

The example, illustrated in figure 4, is an experimental situation in which a subject views a curved instead of flat two-dimensional object. The line connecting points A and B may be thought of as a segment of a circle with center S and radius ρ . If the shape was moved towards or away from the origin O, the contour of the object would also change to maintain a constant value ρ at every position. In other words, distances from points on the AB curve to the position S would be equal, though this value ρ may vary as the object is moved. In this situation, an observer could quite easily identify the object in question. Clearly, no distortion due to slant would be present as the object would never be physically slanted away from the observer. This agrees with the fact that in this case component II (see equation 5) reduces to zero. Along with a good perception of the true shape of the object, the observer would also be able to make relatively good estimates of the location of the form. Both these factors, distance and true shape, are contained in the first component of any object.

When an arbitrary object is translated along the X axis, the equation (see formula 7) defining the changes in the angular height becomes more complex. However, when it is broken down into components, as was done in the static case, certain familiar features arise. The total derivative of the angular height with respect to t contains two separate parts, as did the derivative of the static object. The first component contains true shape and distance information while the distortion due to slant is retained in the second. ..

To again illustrate the idea of a true shape and distance factor, another example in which the value is held constant, shown in figure 5, is now given. In this case, an arbitrary object is moved horizontally behind a

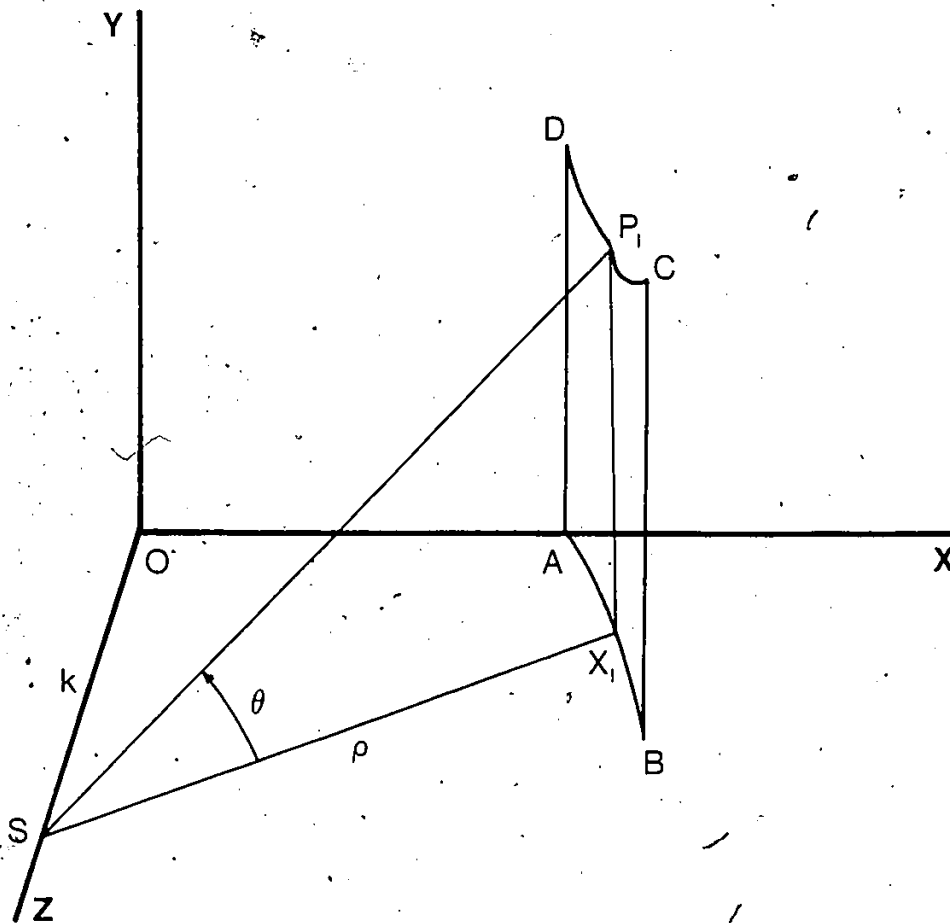


Figure 4. Geometric representation of a curved surface whose base is an arc of a circle with center S .

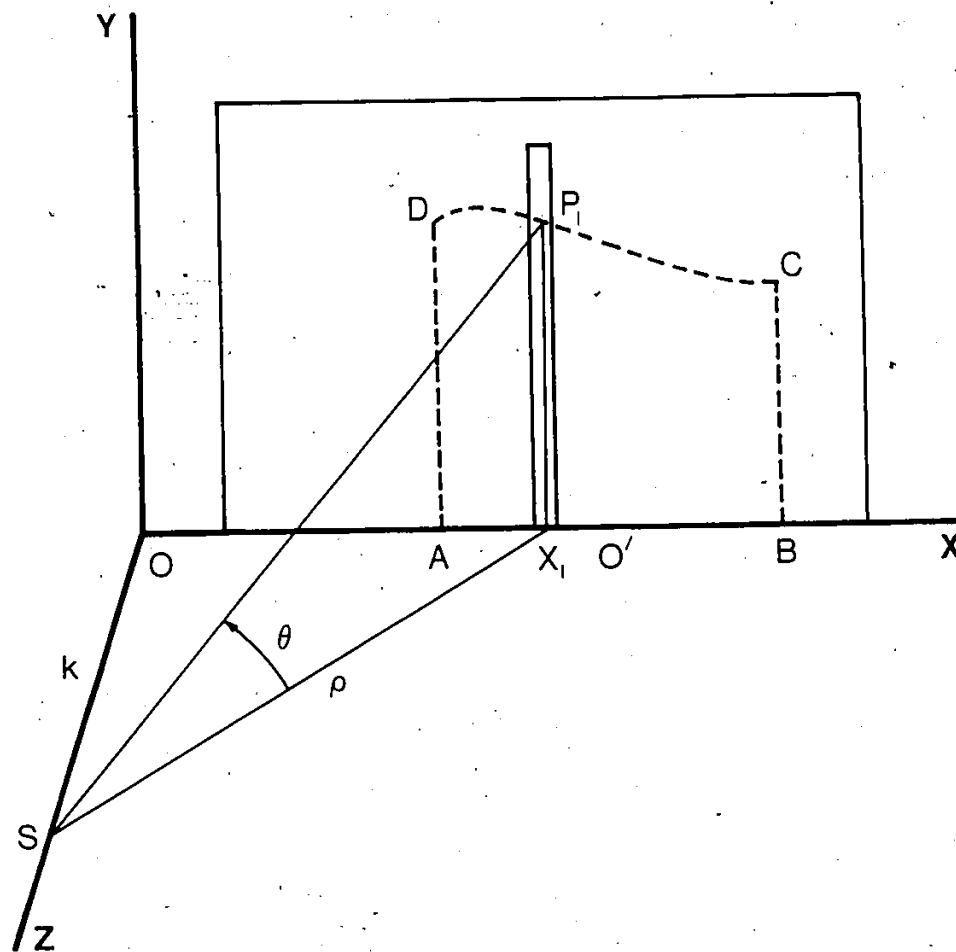


Figure 5. Geometric representation of an arbitrary two-dimensional object viewed through a narrow aperture in a display board.

display board which only has a small narrow vertical aperture in it. The subject may then only perceive the object through this gap in the display board, as the object is translated along the X axis. Here the value ρ is clearly held constant and the angular height depends solely on the shape of the object. Therefore, the observer should receive a true impression of the changes in the angular height without any distortion caused by a changing ρ . Note also that in this example, $du/dt = -dv/dt$ and so component IV = 0. Therefore, $d\theta/dt = \text{component III}$.

This example is similar to one given by Park (1965) in a study of post-retinal visual storage. An apparatus like the one shown in figure 5 was used and an outline of a camel was passed behind it. The observers were not assumed to be fixed in one position however. This difference may be ignored though, since in both cases subjects must view the objects through a narrow aperture, and therefore, slant should not affect the overall perception.

When the figure of the camel passed the opening in the display board completely within approximately $\frac{1}{2}$ seconds, it was seen briefly in the region of the slit, to move slightly and to be foreshortened (Park, 1965). This foreshortening or compression increased with the speed of the figure's passage until only a blur was seen. Park found that at all speeds, except when the outline became a blur, a camel was identified by the subjects. That is, the true shape in terms of height was realized. The compression phenomenon may be explained by noting that the observers can only guess at the width of the object by estimating the rate at which the shape passes the narrow slit in the display board. So, for example, if an underestimate

of the speed of the figure's translation was made, a psychological foreshortening may result.

When comparing the static and moving cases, it is evident that similar information is contained in each. Component I is approximately equivalent to component III, both containing true shape and distance information, except that v is not assumed to be fixed in the latter formula (see equations 4 and 8). The distortion due to slant components II and IV (see equations 5 and 9) corresponding to the static and moving states, respectively, differ by a translation velocity factor (i.e. dv/dt) which is contained in the second.

Hence, overall the only significant difference between the static and moving cases arises from a discrepancy in the velocity terms found in the distortion due to slant components. Therefore, theoretically, recognition should improve when an object is moved, as compared to when it is stationary, provided that the sum of the velocities in component IV amounts to less than the scanning velocity alone, which is found in component II (i.e. $(dv/dt) + (du/dt) < du/dt$). An extreme example of this phenomenon may be found in the Park (1965) experiment, discussed earlier in this paper, in which the velocities cancelled each other out. This, of course, would be an artificial case since the usual everyday perception of an object is not restricted to a small slice of the figure which is seen through a narrow aperture in a display board, as in the Park study.

Two Common Shapes

The rectangle was the first common shape to be studied using the general

formulae developed in the analysis of arbitrary objects. Since the height of the shape in this case is constant, the angular height varied only with changes in the value ρ . Hence, when the object was held stationary and the angular height formula was differentiated, a very simple equation (see formula 11) relating the changes in θ with respect to time t resulted. The first component reduced to zero and changes in the angular height were found to be caused only by the distortion due to slant. In other words, since the form had constant height, no change in the angular height was found to be due to changes in the true shape.

When the rectangle was allowed to move along the X axis and the total derivative of the angular height with respect to time t was taken, the first component again reduced to zero (see equation 12). That is, since the height of the object was constant, even while the rectangle moved horizontally in the observer's frontal plane, changes in the angular height were due only to a distortion due to slant. This distortion, however, depends not only on the changes in u , as in the static case, but also on changes in v with respect to time t .

Differences between the results for the rectangle and trapezoid are quite obvious. The equation (see formula 13) relating changes in angular height for a trapezoid does contain a first component since the object's height is not constant. Also, as the value y does vary, the second component was not found to be equivalent to the corresponding one in the case of a rectangle (see equation 11). Similarly, when the total derivative of angular height was taken with respect to time t , component III did not reduce to zero (see equation 14). Therefore, to perceive a rectangle as such, an observer must

realize that all the distortion in the retinal image is due only to the orientation of the object, whether it is moving or not.

The second common shape to be studied was the circle. Again the general formulae derived in the analysis of arbitrary objects were used to obtain equations (see formulae 15 and 16) relating the changes in the angular height with respect to time t . In the first case, when the circle was assumed to be stationary, the first component did not reduce to zero, as for the arbitrary rectangle. This would, of course, be expected because the circle does not possess a constant height. Note also that the first component changes sign as the subject scans past the center of the circle. This fact can be attributed to the symmetry of the object.

When the circle was allowed to translate horizontally along the X axis, component III in equation 16 was found to be approximately equivalent to component I, as was also noted in the general case. Component IV is also similar to component II, except for the addition translation velocity factor. Hence, the distortion due to slant component varies with respect to du/dt and dv/dt , instead of just du/dt as in the static case. In other words, the difference between the perception of a static or moving circle is that in the latter case, the amount of distortion is changed with the movement of the object. If the distortion due to slant is reduced by motion, better shape recognition should result.

The most obvious difference between the equations derived for the circle and those for the ellipse was the factor, m found in equations 21 and 22. Clearly, as the value of m increases, the discrimination between the two

similar forms improves. When the circle or ellipse is put into motion, this m factor may become more pronounced and discrimination between the shapes would then be enhanced, provided the distortion due to slant factor is reduced.

Conclusions

The analysis showed that the perception of two-dimensional objects presented in the frontal plane may be broken down into two components, whether the object is stationary or moving. The first is a true shape and distance factor, while the second contains distortion due to slant. Borrowing terms from the study of experimental design for illustrative purposes, the first factor may be thought of as containing the two main effects and interaction, and the second the error term. The sum of these two components add up to the actual physical perception or total variance in analysis of variance jargon. To obtain correct perceptions, observers must psychologically separate the first component from the error term. That is, realize the true shape and distance, and ignore the distortion due to slant.

The difference between the static and moving states is a discrepancy in the distortion due to slant components. Only a scanning velocity is involved in the stationary case, while for an object in motion, both the scanning and translation velocities are evident. Hence, an object in a moving state should be perceived better than when in a static position, provided motion reduces the distortion due to slant factor. In other words, recognition or discrimination should improve if the sum of the scanning and translation velocities amounts to less than the scanning velocity alone.

In the specific case of a rectangle, in order to perceive one at a slant, observers must first realize that all the distortion in the retinal image is due to slant only and not from possibly viewing a trapezoid or similar shape. This recognition task may be simplified when the rectangle is translated in the frontal plane if the distortion is decreased, making the true shape become more evident. In the case of a circle, motion may facilitate recognition or discrimination in a similar way. If the distortion due to slant is reduced by moving the object, the true circle or ellipse shape should become more obvious.

The general formulae developed in this study may be applied to actual empirical experiments. Of course, the basic assumptions, such as a fixed observer, would have to be satisfied before the formulae could be utilized. The defining equation of the specific experimental shape being used and other parameters such as speed of object translation would be inserted into the general equations. Information derived from these formulae could then be related to results obtained from empirical recognition experimentation.

The general approach used in this present study may be extended to the analysis of two-dimensional objects being rotated or translated in other planes, or even three-dimensional objects. However, in the future, instead of using the angular height to express physical impressions, an analysis of vectors, extending from an observer to an object, may be preferable. Though this technique could not be related back to the physical characteristics of sight as easily as the angular height may, it would enable an investigator to handle more complex shapes and orientations.

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